The Dynamics of the Airplane. (Mathematical Monographs, No. 21). By KENNETH P. WILLIAMS, Associate professor of mathematics, Indiana University. New York, John Wiley & Sons, 1921. 8vo. 8 + 138 pp. Price $2.50.

Preface: "It was the good fortune of the author to attend the University of Paris during the spring semester of 1919. One of the special courses which the French authorities, with their characteristic hospitality, arranged for the large number of students from the American army, was a course in aërodynamics, given by Professor Marchis. The comprehensive knowledge that Professor Marchis possessed of all branches of the new science of aëronautics, the inestimable value of his advice to the French Republic during the war, the interest he took in his rather unusual class, could not fail to be an inspiration.

"This book is an outgrowth of those parts of Professor Marchis' lectures that were of particular interest to the author. It is in no sense a complete treatise on aviation. Questions of design and construction are passed over with bare mention. The book is intended for students of mathematics and physics who are attracted by the dynamical aspect of aviation. The problems presented by the motion of an airplane are novel and fascinating. They vary from the most pleasing simplicity to the most stimulating difficulty. The question of stability, particularly, exhibits at the same time the elegance and the power of analysis, and shows the adaptability of some of the general developments in dynamics. The field is assuredly a fruitful one of study, and increasing demands will be put upon the mathematician as the science of aviation continues its rapid development. The mathematician can well own a sense of pride that he had already at hand, in the developments inaugurated by Euler and Routh, a means of dealing accurately with the question of stability, that plays so fundamental a rôle in the science of flying.

"The treatment in the text is for the most part elementary. The last chapter alone demands of the student familiarity with more advanced dynamical methods. In the treatment of descent a slight digression is made to consider in part the nature of the solution of a system of two differential equations. This was done in order not to completely evade what seems a problem of considerable difficulty. It might seem that a treatment of the propeller should not find a place in a book with the purpose of this one. No student of mathematics, however, could fail to own a curiosity as to a propeller's action, and it is hoped the discussion, while not complete, will at least serve as a sufficient introduction."

Contents—

NOTES.

It is announced that John Wallis's *Arithmetica Infinitorum*, translated and edited by J. M. Child, is soon to be published by The Open Court Company, London—The second edition of Hobson's *The Theory of Functions of a Real Variable* is to be in two large volumes. The first volume appeared early in this year (Cambridge University Press; 16 + 671 pages; price 45 shillings).

At the meeting of the Academy of Sciences of the Institute of France last December, several sums of money from the Loutreuil Foundation were voted to assist scientific publications. Among them was 3000 francs to T. Lemoyne and H. Brocard "for the publication of the second and third volumes of their work *Courbes géométriques remarquables* (*courbes spéciales*) *planes et gauches*" (*1920*, 130).

The MONTHLY draws the attention of mathematicians to the excessive charges for publications of the Cambridge University Press made by the present American agent, The Macmillan Company. A single illustration will suffice.

The third edition of Whittaker and Watson's *A Course of Modern Analysis* was published at 40 shillings (*1921*, 31); the American agent's price is $12.50. Hence by ordering from a London bookseller, and paying the duty, a saving of at least $3.00 on the purchase of this single volume could be effected.—There is no duty for books ordered for college and public libraries. The American Branch of the Oxford University Press appears to count more definitely on the ignorance of purchasers in the United States. To illustrate: H. Hilton's *Plane Algebraic Curves*, 1920, was published at 28 shillings (about $5.60 at the present rate of exchange); the price of the American Branch is $12.60!

The second and concluding number of *Mathematische Annalen*, volume 80, is to be a Generalregister of volumes 51–80. The Generalregister, volumes 1–50 (11 + 202 pp.), was published in 1898.

Julius Springer, of Berlin, published early in 1921, the first volume (price 186 marks, "Zuschlagfrei," 12 + 612 pages + portrait) of Felix Klein's *Gesammelte Mathematische Abhandlungen*, "herausgegeben von R. FRICKE und A. OSTROWSKI (von F. Klein mit ergänzenden Zusätzen versehen)." The volume contains: "Liniengeometrie, Grundlegung der Geometrie zum Erlanger Programm."

In *Jahresbericht der deutschen Mathematiker-Vereinigung*, volume 29, nos. 1–6, September, 1920, we find (pp. 28–40) "Ein Beitrag zur Lebensbeschreibung von L. Fuchs" by L. SCHLESINGER, who has for years been collecting material for a life of FUCHS. Schlesinger refers to LEO KÖNIGSBERGER'S book, *Mein Leben* (Heidelberg, 1919), where there are various "assertions and judgments concerning L. Fuchs which do not in any wise correspond to the picture which I bear in my heart of my unforgettable teacher, and fatherly friend." In support of his contentions Schlesinger publishes several letters written by Weierstrass in 1875.

We have already referred (*1921*, 32) to the first of a series of memoirs by G. H. HARDY and J. E. LITTLEWOOD, on "Some problems of 'partitio numerorum'" published in the Göttingen *Nachrichten*, 1920, pp. 33–54. The second memoir was published January 31, 1921, in *Mathematische Zeitschrift*, volume 9, nos. 1–2, pp. 14–27, and deals with the "Proof that every large number is the sum of at most 21 biquadrates." The same issue of the *Zeitschrift* contains an article on "Congruence properties of partitions" by S. Ramanujan (see *1920*, 338). This is extracted by G. H. Hardy from one of the author's manuscripts which "contains a large number of further results. It is very incomplete, and will require very careful editing before it can be published in full. I have taken from it," Hardy writes, "the three simplest and most striking results, as a short but characteristic example of the work of a man who was beyond question one of the most remarkable mathematicians of his time."

Nature for February 17, 1921, was a special number on relativity, and contained the following articles, pages 782–811: "A brief outline of the development

of the theory of relativity" by A. EINSTEIN; "Relativity: the growth of an idea" by E. CUNNINGHAM; "Relativity and the eclipse observations of May, 1919" by F. DYSON; "Relativity and the motion of Mercury's perihelion" by A. C. D. CROMMELIN; "The displacement of solar lines" by C. E. ST. JOHN; "Non-Euclidean geometries" by G. B. MATHEWS; "The general physical theory of relativity" by J. H. JEANS; "The Michelson-Morley experiment and the dimensions of moving bodies" by H. A. LORENTZ; "The geometrisation of physics, and its supposed basis on the Michelson-Morley experiment" by O. LODGE; "Electricity and gravitation" by H. WEYL; "The relativity of time" by A. S. EDDINGTON; "Theory and experiment in relativity" by N. CAMPBELL; "The relation between Geometry and Einstein's theory of gravitation" by DOROTHY WRINCH and H. JEFFREYS; "The metaphysical aspects of relativity" by H. W. CARR. The "Bibliography of relativity" (pages 811–813) lists (1) 83 books and pamphlets (among the names of authors are: F. S. WOODS, E. V. HUNTINGTON, R. D. CARMICHAEL, L. SILBERSTEIN, R. C. TOLMAN and E. E. SLOSSON); and (2) the articles on the subject, which have appeared in *Nature*. These were mainly selected from a bibliography of about 650 titles for the period 1886–1920, compiled by H. F. Morley, director of the *International Catalogue of Scientific Literature*.—This special number of *Nature* was in great demand and was entirely out of print on March 17.

A new part of the great *New English Dictionary on Historical Principles*, Visor-Vywer, was issued from the Clarendon Press in March, 1920 (see *1919*, 256–257, *1920*, 128–129). The words in the section of special interest to the mathematician are: viss, volume and vortex (and their derived words and compounds), vorty, vulcan, vulgar fractions, and vulpecula—Viss is a weight used in southern India and Burmah equal to about three and one half pounds; it has appeared in English publications since 1588—Of the article on volume, paragraphs 8–9 give illustrations of the use of the word since 1792 as "the bulk, size, or dimensions of a thing," "the mass or solid body of something," "the amount or quantity of something," and "without article: bulk, mass, dimensions." Volumenometer, volumometer, volumenometry and volumetry are illustrations of derived words—Vortex has been used since its introduction in older theories of the universe, especially that of Descartes. Derived words are vorticose, vorticular and vortiginous—Vorty, a "south-western dialect form for forty," occurs in seventeenth century literature—Vulcan a hypothetical planet supposed to have its orbit between the sun and mercury—The earliest quotation for the term "vulgar fractions" is in Jeake's *Arithmetic*, 1696. The obsolete expression "vulgar arithmetic" was employed in the seventeenth and eighteenth centuries —Vulpecula is a small northern constellation lying between Hercules and Pegasus.

Among universities of America which have recently published complete lists of those on whom the degree of doctor of philosophy, with mathematics as a major, has been conferred are: (*a*) The University of Chicago, (*b*) Harvard University, and (*c*) The Johns Hopkins University.

(*a*) This list is in *Circular of the Departments of Mathematics, Astronomy and Astrophysics, Physics, Chemistry, 1920*. There have been 85 mathematical doctors, 1896–1919, L. E. Dickson and J. I. Hutchinson being the first. With mathematical astronomy as a major, there were 11 more, 1900–1915. No indication is given as to whether or not the dissertations have been published, although it is known that this occurred in most cases.

(*b*) This list is to be found in *Official Register of Harvard University, Division of Mathematics, 1920–1921*. There were 47 doctors in mathematics, 1873–1919, the first being W. E. Byerly. Indications are given of the place of publication of 32 of the dissertations.

(*c*) *Doctors' Dissertations 1878–1919* is the title of *The Johns Hopkins University Circular*, new series, 1920, no. 1. In mathematics there were 69 doctors, 1878–1918, the first being Thomas Craig; in astronomy there were 5, 1891–1898. Indications are given regarding the publication of all of the dissertations except for 14 in mathematics and 2 in astronomy. Of these, 24 were published in the *American Journal of Mathematics*.

Since 1896, according to the above mentioned lists for mathematics, Harvard has had 41 doctors and Johns Hopkins, 46; so that during the period 1896–1919 the University of Chicago has conferred almost as many of such doctorates as Harvard and Johns Hopkins together.

The following extracts are from John Dewey's *Reconstruction in Philosophy* (New York, Holt, 1920).

"Mathematics is often cited as an example of purely normative thinking dependent upon *a priori* canons and supra-empirical material. But it is hard to see how the student who approaches the matter historically can avoid the conclusion that the status of mathematics is as empirical as that of metallurgy. Men began with counting and measuring things just as they began with pounding and burning them. One thing, as common speech profoundly has it, led to another. Certain ways were successful—not merely in the immediately practical sense, but in the sense of being interesting, of arousing attention, of exciting attempts at improvement. The present-day mathematical logician may present the structure of mathematics as if it had sprung all at once from the brain of a Zeus whose anatomy is that of pure logic. But, nevertheless, this very structure is a product of long historic growth, in which all kinds of experiments have been tried, in which some men have struck out in this direction and some in that, and in which some exercises and operations have resulted in confusion and others in triumphant clarifications and fruitful growths; a history in which matter and methods have been constantly selected and worked over on the basis of empirical success and failure.

"The structure of alleged normative *a priori* mathematics is in truth the crowned result of ages of toilsome experience. The metallurgist who should write on the most highly developed method of dealing with ores would not, in truth, proceed any differently. He too selects, refines, and organizes the methods which in the past have been found to yield the maximum of achievement. Logic is a matter of profound human importance precisely because it is empirically founded and experimentally applied. So considered, the problem of logical theory is none other than the problem of the possibility of the development and employment of intelligent method in inquiries concerned with deliberate reconstruction of experience. And it is only saying again in more specific form what has been said in general form to add that while such a logic has been developed in respect to mathematics and physical science, intelligent method, logic, is still far to seek in moral and political affairs" (pages 137–138). "Such a deductive science as mathematics represents the perfecting of method. That a method to those concerned with it should present itself as an end on its own account is no more surprising than that there should be a distinct business for making any tool. Rarely are those who invent and perfect a tool those who employ it. There is,

indeed, one marked difference between the physical and the intellectual instrumentality. The development of the latter runs far beyond any immediately visible use. The artistic interest in perfecting the method by itself is strong—as the utensils of civilization may themselves become works of finest art. But from the practical standpoint this difference shows that the advantage as an instrumentality is on the side of the intellectual tool. Just because it is not formed with a special application in mind, because it is a highly generalized tool, it is the more flexible in adaptation to unforeseen uses. It can be employed in dealing with problems that were not anticipated. The mind is prepared in advance for all sorts of intellectual emergencies, and when the new problem occurs it does not have to wait till it can get a special instrument ready" (page 149).

ARTICLES IN CURRENT PERIODICALS.

American Oxonian, Concord, N. H., volume 8, no. 1, January, 1921: "The record of the American Rhodes Scholars" by R. W. Burgess, 1–36 [Of the 351 American Rhodes Scholars at Oxford, classes matriculating 1904–1914, 32.7 percent studied law; 17.1 percent modern history and economics; 16.8 percent humanities, including the classics, philosophy, and anthropology; and 6 percent mathematics, physics, chemistry and engineering].

Bulletin of the American Mathematical Society, volume 27, no. 4, January, 1921: "The October meeting of the San Francisco Section" by B. A. Bernstein, 149–153; "An image in four-dimensional lattice space of the theory of the elliptic theta functions" by E. T. Bell, 153–160; "Note on the median of a set of numbers" by D. Jackson, 160–164; "Note on closure of orthogonal sets" by O. D. Kellogg, 165–169; "The mathematical work of Thomas Jan Stieltjes" [review of *Oeuvres complètes de Thomas Jan Stieltjes* (2 volumes, Groningen, 1914–1918)] by R. D. Carmichael, 170–178; Reviews by D. E. Smith of *Opere di Evangelista Torricelli* (edited by G. Loria and G. Vassura, 2 volumes, Faenza, 1919) and of W. W. R. Ball's *An Introduction to String Figures* (Cambridge, 1920), 178–181; Review by C. N. Moore of A. R. Forsyth's *Solutions of the Examples in a Treatise on Differential Equations* (London, 1918), 181–182; Reviews by E. B. Wilson of A. S. Eddington's *Space, Time, and Gravitation; an Outline of the General Relativity Theory* (Cambridge, 1920) and of R. W. Wood's *Researches in Physical Optics* (New York, 1919), 182–186; Notes, 187–193; New publications, 194–196.

Educational Administration and Supervision, volume 6, no. 9, December, 1920: "The scoring of geometry test W" by J. H. Minnick, 509–511.

Hibbert Journal, volume 19, January, 1921: Review by C. D. Broad of A. N. Whitehead's *The Concept of Nature* (Cambridge, 1920), 360–366 [Last paragraph: "The thanks rendered in the preface by Professor Whitehead to the Cambridge University Press officials seem to me excessive. No doubt their hearts are in the right place, but they have passed at least six bad mistakes. On p. 51, l. 4, for *sight* read *touch;* p. 86, l. 8, for *external, eternal;* p. 148, l. 4, for *agree, argue;* p. 155, l. 17, for *sense-object, perceptual object;* p. 180, l. 4, for *universely* (a pleasant conceit!), *inversely;* and on p. 188, l. 9, for *by* read *from.* In conclusion, I must say that anyone who has read *Principles of Natural Knowledge* will find his understanding of that book much improved by reading *The Concept of Nature;* and that anyone who has read neither should go at once to his (or her) bookseller and order both."]

Journal of Philosophy, volume 28, no. 2, January 20, 1921: "Eddington on Einstein" by E. E. Slosson, 48–51 [Last paragraph: "Some mathematicians and physicists have manifested impatience at the impertinent curiosity of the public and declare that Einstein's theory concerns only themselves, and whatever they may decide to do with it can have no possible effect upon anybody's religion, philosophy or view of life. But the public knows better. And Professor Eddington agrees with the majority on this question. Galileo, Newton and Darwin were specialists, speculating in fields remote from common life, yet they have revolutionized the thought and altered the conduct of the world. Einstein's theory is even more fundamental and uncontentional and if it is verified by experiment or generally adopted as a working hypothesis it will be found in the course of time to have a profound influence upon the minds of men outside of the realm of science."]

Journal of the United States Artillery, volume 51, December, 1919: "Charts for the calculation of the effect of small changes in the elements of fire" by P. L. Alger, 585–603—Volume 53, August, 1920: "Two misconceptions" by R. S. Hoar, 179–181—October: "On weighting factor curves for flat fire" by J. F. Ritt, 404–410—December: "Wind weighting factors" by J. J. Johnson, 578–587; "Mirror and window position finders" by W. C. Graustein, 588–610.